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**Weighted estimates for a class of matrix operators in sequence space**

**ABSTRACT**
**of the dissertation work for degree of Doctor of Philosophy  (PhD) in specialty**
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**The relevance of the topic.** Various properties of discrete operators play an important role in many branches of mathematics, including function theory, harmonic analysis, the theory of difference equations, the theory of spaces involving difference operators, and the spectral theory of difference operators. In the theory of matrix operators, particular attention is given to the issues of boundedness, compactness, and norm estimation in different sequence spaces. However, in several areas of theoretical and applied significance, these problems remain unsolved. for instance, the boundedness of a general matrix operator mapping one Lebesgue sequence space into another for $1 < p, q < \infty$ is still an open question. Therefore, it is essential to identify certain classes of matrix operators that arise in solving various analytical problems and to study their properties.

The development of the theory of matrix operators originated from the study of weighted estimates for Hardy-type operators and their generalizations. fundamental results in this direction were obtained by G. Hardy, J. E. Littlewood, G. Pólya, B. Opic, A. Kufner, L-E. Persson, and others. Later studies have extensively investigated classes of matrices satisfying specific properties, particularly the class known as "Oinarov kernels". The elements of such matrices satisfy a discrete condition - "Oinarov's condition" which frequently appears in Volterra-type integral operators. An integral operator whose kernel satisfies Oinarov's condition encompasses a wide class of fractional integration operators and provides criteria for boundedness and compactness of operators between various functional spaces.

One of the significant modern developments in this field is the introduction by Oinarov of the classes of matrices $O_n^\pm$, for which necessary and sufficient conditions for boundedness and compactness in weighted Lebesgue sequence spaces have been established. However, for $1 < q < p < \infty$, the problem for operators belonging to $O_n^\pm, n \geq 2$ remains open.

This dissertation is devoted to weighted estimates of certain classes of matrix operators defined in weighted Lebesgue sequence spaces. The research focuses on one of the key problems in the theory of matrix operators — establishing criteria for their boundedness and compactness and studying weighted estimates of matrix operators on sets of monotone sequences.

**Aim of the research.** To determine necessary and sufficient conditions for the validity of weighted inequalities for certain classes of matrix operators in Lebesgue sequence spaces. The study considers matrix operators belonging to Oinarov's class

$O_2^{\pm}$, operators with variable summation limits, and discrete Hilbert–Stieltjes-type operators.

**Research objectives:**
- for $1 < q < p < \infty$, to obtain necessary and sufficient conditions for boundedness of matrix operators of class $O_2^{\pm}$ in weighted Lebesgue sequence spaces in terms of matrix elements and weight sequences;
- to establish necessary and sufficient conditions for weighted estimates of matrix operators with variable summation limits in Lebesgue sequence spaces;
- to study weighted estimates of matrix operators with variable summation limits on sets of monotone sequences;
- to formulate boundedness and compactness criteria for such matrix operators.

**Object of the study.** The class of matrix operators in weighted Lebesgue sequence spaces.

**Research methods**. Localization method, the Batuev–Stepanov block-diagonal method, weighted hardy inequalities, and classical analytical techniques were used.

**Scientific novelty**. The dissertation examines the following discrete operators: matrix operators of Oinarov's class $O_2^{\pm}$, matrix operators with variable summation limits, and discrete Hilbert–Stieltjes-type operators.

The obtained results include:
- in terms of matrix elements and weight sequences, necessary and sufficient conditions for the boundedness of a matrix operator from the class $O_2^{\pm}$ in weighted Lebesgue spaces of sequences for $1 < q < p < \infty$;
- in terms of matrix elements and weight sequences, necessary and sufficient conditions for the boundedness of a matrix operator with variable summation limits in weighted Lebesgue spaces of sequences for case $1 < p \leq q < \infty$:
- criteria for compactness of a matrix operator with variable summation limits in weighted Lebesgue spaces of sequences for the case $1 < p \leq q < \infty$ ;
- weighted estimate of a matrix operator with variable summation limits on a set of monotone sequences;
- weighted estimates of a discrete Hilbert–Stieltjes type operator in the space of Lebesgue sequences for $1 < p, q < \infty$.

**Theoretical and practical significance**. The research is theoretical in nature. Its results can be applied in function theory, the theory of discrete weighted Sobolev-type spaces, and the theory of difference operators.

**Approbation of the results.** The main results were presented at international conferences and scientific seminars.

- International Scientific Conference "Actual Problems of Analysis, Differential Equations, and Algebra" (EMJ-2019), Nur-Sultan, 2019;
- International Scientific Conference "Theoretical and Applied Issues of Mathematics, Mechanics and Computer Science", Karaganda, 2019;
- International April Mathematical Conference in Honor of the Day of Scientists of the Republic of Kazakhstan, Almaty, 2020;

- Republican Scientific Conference with the Participation of Foreign Scientists "Modern Methods of Mathematical Physics and Their Applications", Tashkent, Uzbekistan, 2020;

- XI International Conference of the Georgian Mathematical Union Batumi, Georgia, Batumi, 2021;

- Traditional International April Mathematical Conference in Honor of the Science Day of the Republic of Kazakhstan, Almaty 2023;

- International Scientific and Practical Conference "Analysis, Differential Equations and Their Applications", Astana, 2023;

- VII World Congress of Mathematicians of the Turkic World (TWMS Congress-2023), September 20–23, 2023, Turkestan, Kazakhstan;

- International Conference "Modern Problems of Mathematics, Mechanics and Their Applications", June 20-22, 2024, Baku, Azerbaijan;

- International Conference on Mathematics and Mathematics Education (ICMME-2024), Nevsehir Hacı Bektaş Veli University, Cappadocia University, Nevsehir, Türkiye, October 3–5, 2024;

**Publications**. The results of the dissertation were published in 5 works, including 2 articles in a top-ranked journal indexed in the Scopus and Web of Science databases with a CiteScore $\geq 25\%$, 2 articles in publications recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan and 9 conference abstracts were also published, including 3 foreign ones.

**Structure of the dissertation**. The dissertation consists of an introduction, three chapters, a conclusion, and a bibliography. Each section is divided into subsections.

The formulas are numbered using three indices. The first index indicates the section number, the second index indicates the order of the section's subsections, and the third index indicates the order of the formulas within that subsection. The dissertation is 94 pages long.

**Number of sources used** – 88.

**Keywords**. Weighted Lebesgue space, discrete operator, matrix operator, Hardy-type inequality, monotone sequence.